Lightcone fluctuations in quantum gravity and extra dimensions

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Abstract

We discuss how compactified extra dimensions may have potentially observable effects which grow as the compactification scale decreases. This arises because of lightcone fluctuations in the uncompactified dimensions which can result in the broadening of the spectral lines from distant sources. We analyze this effect in a five dimensional model, and argue that data from gamma ray burst sources require the compactification length to be greater than about 10^5 cm in this model.

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One of the most challenging problems in modern physics is the unification of the gravitational interaction with other known interactions in nature. Many attempts involve going to higher dimensions and postulating the existence of extra spatial dimensions. If these extra dimensions really exist, one must explain why they are not seen. The usual answer is that they curl into an extremely small compactified manifold, possibly as small as the Planck length, $l_{pl} = 1.6 \times 10^{-33}$ cm. Therefore low-energy physics should be insensitive to them until distances of the compactification scale are being probed. In general, one has the possibility of observing the presence of the extra dimensions in a scattering experiment in which energies greater than that associated with the compactification scale are achieved. Many extra dimension models are constained to have extra dimensions no larger than about $(1\text{Tev})^{-1} = 2 \times 10^{-17}$cm. However, if only gravity propagates in the extra dimensions, the upper bound can be much larger. A recent proposal is that the fundamental scale of quantum gravity can be as low as few Tev and the observed weakness of gravity is the result of large extra dimensions in which only gravity can propagate [1–3]. The size of extra dimensions (of which there must be at least two) can be as large as 1 mm in this type of model.

However, a question arises naturally as to whether there are any lower bounds on the sizes of extra dimensions. It is the common belief that the existence of extra dimensions has no effect on low-energy physics as long as they are extremely small. We will argue in this letter that this need not be the case. The reason is lightcone fluctuations arising from the quantum gravitational vacuum fluctuations, due to compactification of spatial dimensions [4,5]. The compactification of spatial dimensions gives rise to stochastic fluctuations in the apparent speed of light which are in principle observable. Basically, the smaller the size of the compactified dimensions , the larger are the fluctuations that result. This is closely related to the Casimir effect, the vacuum energy occuring whenever boundary conditions are imposed on a quantum field. The gravitational Casimir energy in the five-dimensional case with one compactified spatial dimension was studied in [6], where a nonzero energy density was found, which tends to make the extra dimension contract. This raises the question of stability of the extra dimensions. It is possible, however, that the Casimir energy arising from the quantum gravitational field and other matter fields may be made to cancel each other [7,8], thus stabilizing the extra dimensions. Quantum lightcone fluctuations due to the compactfication of spatial dimensions [4], although similar in nature to the Casimir effect, come solely from gravitons. Hence, no similar cancelation is to be expected.

Note that all of these quantum effects increase in magnitude as the compactification scale decreases. The effect of compatification in, for example, a five dimensional model can be thought of as producing an infinite tower of massive modes with masses inversely proportional to the compactification scale. One might think that the contribution of these modes to radiative correction would decrease with decreasing compactification scale, whereas in fact the reverse is actually the case. Detailed calculations of the vacuum polarization [9] and of the electron self-energy [10] have been performed which reveal growing effects with decreasing compactification length. This can be understood as a consequence of the uncertainty principle; the fluctuations of quantum fields confined in a finite region increase as the size of the region decreases.

In a recent work, we studied the light cone fluctuations in four dimensional flat spacetime with compactification in one spatial dimension [4]. It was found that these fluctuations, although typically of the order of the Planck scale, can get larger for path lengths

large compared to the compactification scale. In particular, the mean deviation from the classical propagation time, Δt, is proportional to the square root of the travel distance, r. In this paper we apply the formalism to spacetimes with extra dimensions, and examine, in particular, the five dimensional case to demonstrate a possible observable consequence of compactification of the extra dimension. To begin, let us examine a d dimensional spacetime with $d-4$ extra dimensions. Consider a flat background spacetime with a linearized perturbation $h_{\mu\nu}$ propagating upon it , so the spacetime metric may be written as $ds^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu = dt^2 - d\mathbf{x}^2 + h_{\mu\nu}dx^\mu dx^\nu$, where the indices μ, ν run through $0, 1, 2, 3, ..., d-1$. Let $\sigma(x, x')$ be one half of the squared geodesic distance between a pair of spacetime points x and x', and $\sigma_0(x, x')$ be the corresponding quantity in the flat background. In the presence of a linearized metric perturbation $h_{\mu\nu}$, we may expand $\sigma = \sigma_0 + \sigma_1 + O(h_{\mu\nu}^2)$. Here σ_1 is first order in $h_{\mu\nu}$. If we quantize $h_{\mu\nu}$, then quantum gravitational vacuum fluctuations will lead to fluctuations in the geodesic separation, and therefore induce lightcone fluctuations. In particular, we have $\langle \sigma_1^2 \rangle \neq 0$, since σ_1 becomes a quantum operator when the metric perturbations are quantized. The quantum lightcone fluctuations give rise to stochastic fluctuations in the speed of light, which may produce an observable time delay or advance Δt in the arrival times of pulses. Note that this model uses a linearized approach to quantum gravity which is expected to be a limit of a more exact theory. In the absence of a full theory, this seems to be the most conservative way to compute quantum gravity effects. One might comtemplate doing a one-loop calculation of an S-matrix element, along the lines of those in Refs. [9,10] for electrodynamics. However, one would need to find a way to deal with the nonrenormalizability of one-loop quantum gravity coupled to other fields [11].

Let us consider the propagation of light pulses between a source and a detector separated by a distance r on a flat background with quantized linear perturbations. In Ref. [5] it was shown that the root-mean-squared fluctuation in the propagation time is

$$\Delta t = \frac{\sqrt{\langle \sigma_1^2 \rangle_R}}{r} , \tag{1}$$

where $\langle \sigma_1^2 \rangle_R$ is a renormalized expectation value, which was assumed to be positive. We can give an alternative derivation which applies in the case $\langle \sigma_1^2 \rangle_R < 0$. For a pulse which is delayed or advanced by time Δt, which is much less than r, one finds

$$\sigma = \sigma_0 + \sigma_1 + ... = \frac{1}{2}[(r + \Delta t)^2 - r^2] \approx r\Delta t\,. \tag{2}$$

Take the fourth power of the above equation and average over a given quantum state of gravitons $|\phi\rangle$ (e.g. the vacuum states associated with compactification of spatial dimensions),

$$\Delta t^4 = \frac{\langle \phi | \sigma_1^4 | \phi \rangle}{r^4}\,. \tag{3}$$

This result is, however, divergent due to the formal divergence of $\langle \phi | \sigma_1^4 | \phi \rangle$. We can define $\langle \phi | \sigma_1^4 | \phi \rangle$ by normal ordering, and let $\langle \phi | \sigma_1^4 | \phi \rangle_R = \langle \phi | : \sigma_1^4 : | \phi \rangle$. For a free field ψ, Wick's theorem yields $\langle : \psi^4 : \rangle = 3\langle : \psi^2 : \rangle^2 = 3\langle \psi^2 \rangle_R^2$, where the expectation value is in the vacuum state. Hence a suitable meaure of the deviation from the classical propagation time is given by

$$\Delta t = \frac{(3\langle\sigma_1^4\rangle_R)^{1/4}}{r} = \frac{3^{1/4}\sqrt{|\langle\sigma_1^2\rangle_R|}}{r} \approx \frac{\sqrt{|\langle\sigma_1^2\rangle_R|}}{r}\,. \tag{4}$$

Apart from small numerical factors which we ignore, this is equivalent to replacing $\langle\sigma_1^2\rangle_R$ by $|\langle\sigma_1^2\rangle_R|$ in Eq. (1). The gauge invariance of this expression has been analyzed recently [4].

Note, however, that Δt is the ensemble averaged deviation, not necessarily the expected variation in flight time, δt, of two pulses emitted close together in time. The latter is given by Δt only when two successive pulses are uncorrelated. This point is discussed in detail in Ref. [12]. These stochastic fluctuations in the apparent velocity of light arising from quantum gravitational fluctuations are in principle observable, since they may lead to a spread in the arrival times of pulses from distant astrophysical sources, or the broadening of the spectral lines. This can be used to place a lower bound on the size of the extra dimension. Lightcone fluctuations and their possible astrophysical observability have been recently discussed in a somewhat different framework in Refs. [13–15].

In order to find Δt in a particular situation, we need to calculate the quantum expectation value $\langle\sigma_1^2\rangle_R$ in any chosen quantum state, which can be shown to be given by [4,5]

$$\langle\sigma_1^2\rangle_R = \frac{1}{8}(\Delta r)^2 \int_{r_0}^{r_1} dr \int_{r_0}^{r_1} dr'\; n^\mu n^\nu n^\rho n^\sigma\; G^{(1)R}_{\mu\nu\rho\sigma}(x,x')\,. \tag{5}$$

Here $dr = |d\mathbf{x}|$, $\Delta r = r_1 - r_0$ and $n^\mu = dx^\mu/dr$. The integration is taken along the null geodesic connecting two points x and x', and $G^{(1)R}_{\mu\nu\rho\sigma}(x,x')$ is the graviton Hadamard function, understood to be suitably renormalized. We shall now work in the transverse-tracefree gauge in which the gravitational perturbations have only spatial components h_{ij}. This is a gauge which retains only physical degrees of freedom. The quantized field operator may be expanded as

$$h_{ij} = \sum_{\mathbf{k},\lambda} [a_{\mathbf{k},\lambda} e_{ij}(\mathbf{k},\lambda) f_{\mathbf{k}} + H.c.]. \tag{6}$$

Here H.c. denotes the Hermitian conjugate, λ labels the $\frac{1}{2}(d^2-3d)$ independent polarization states, $f_{\mathbf{k}}$ is the mode function, and the $e_{\mu\nu}(\mathbf{k},\lambda)$ are polarization tensors. (Units in which $32\pi G_d = 1$, where G_d is Newton's constant in d dimensions and in which $\hbar = c = 1$ will be used in this paper.) Now suppose the $(d-1)$-th dimension is compactified into a small size of periodicity length L, so the mode function is given by $f_{\mathbf{k}} = (2\omega(2\pi)^{d-2}L)^{-\frac{1}{2}} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$ with $k_{d-1} = \frac{2\pi n}{L}, n = 0, \pm1, \pm2, \pm3, \ldots$. Let us denote the associated vacuum state by $|0_L\rangle$. In order to calculate the gravitational vacuum fluctuations due to compactification of the extra dimension, we need the renormalized graviton Hadamard function with respect to the vacuum state $|0_L\rangle$, $G^{(1)R}_{ijkl}(x,x')$, which can be seen to be given by an image sum of the corresponding Hadamard function for the uncompactified Minkowski vacuum, $G^{(1)}_{ijkl}$:

$$G^{(1)R}_{ijkl}(t, x_{d-1}, t', x'_{d-1}) = \sum_{n=-\infty}^{+\infty}{}' \, G^{(1)}_{ijkl}(t, x_{d-1}, t', x'_{d-1} + nL)\,, \tag{7}$$

where the prime on the summation indicates that the $n = 0$ term is excluded and the notation $(t, x_1, .., x_{d-2}, x_{d-1}) \equiv (t, x_{d-1})$ has been adopted.

From now on, we will restrict ourselves to the five dimensional case with one compactified dimension. Because the model discussed in this paper has only one extra dimension, the results do not bear directly on the large extra dimension models [1,2], which require at least two extra dimensions. Higher dimensional cases will be discussed in detail elsewhere [16]. To see how light cone fluctuations arise in the usual uncompactified space as a result of compactification of the extra dimension, let us consider a light ray traveling along the x_1 direction from point a to point b, which is perpendicular to the direction of compactification. The relevant graviton two-point function is G_{xxxx}, which can be shown to be given by

$$
\begin{aligned}
G_{xxxx}(t,x_4,\,t',x_4') = 2[&D(t,x_4,\,t',x_4') - 2F_{xx}(t,x_4,\,t',x_4') \\
&+H_{xxxx}(t,x_4,\,t',x_4')]\,.
\end{aligned} \tag{8}
$$

Here $D(x,x')$, $F_{ij}(x,x')$ and $H_{ijkl}(x,x')$ are functions given by.

$$
\begin{aligned}
D(x,x') &= \frac{Re}{(2\pi)^4}\int \frac{d^4\mathbf{k}}{2\omega} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} e^{-i\omega(t-t')} \\
&= \frac{1}{8\pi^2}\frac{1}{(R^2-\Delta t^2)^{(3/2)}}\,,
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
F_{ij}(x,x') &= \frac{Re}{(2\pi)^4}\partial_i\partial_j'\int \frac{d^4\mathbf{k}}{2\omega^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} e^{-i\omega(t-t')} \\
&= \frac{1}{8\pi^2}\partial_i\partial_j'\left(\frac{\sqrt{R^2-\Delta t^2}}{R^2}\right)\,,
\end{aligned} \tag{10}
$$

and

$$
\begin{aligned}
H_{ijkl}(x,x') &= \frac{Re}{(2\pi)^4}\partial_i\partial_j'\partial_k\partial_l'\int \frac{d^4\mathbf{k}}{2\omega^5} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} e^{-i\omega(t-t')} \\
&= 0\,.
\end{aligned} \tag{11}
$$

Here $R = |\mathbf{x}-\mathbf{x}'|$ and $\Delta t = t - t'$.

Inserting the above results into Eq. (5), carrying out the differentiation in the function $F_{x_1x_1}$, using the fact that $x_1 - x_1' = \Delta t$, and then performing the integration, we finally find

$$
\langle\sigma_1^2\rangle_R = \frac{r^2}{32\pi^2 L}\sum_{n=1}^{\infty}\left[\frac{8}{n}\ln(1+\frac{\rho^2}{n^2}) - \frac{2\rho^2}{n^3} - \frac{8\rho^2}{(\rho^2+n^2)n}\right]\,. \tag{12}
$$

Here we have defined $r = a - b$ and a dimensionless parameter $\rho = r/L$. We are interested in the case in which $\rho \gg 1$. It then follows that the summation is dominated, to the leading order, by the second term,

$$
\langle\sigma_1^2\rangle_R \approx -\frac{r^2}{16\pi^2 L}\sum_{n=1}^{\infty}\frac{\rho^2}{n^3} = -\frac{\zeta(3)r^2\rho^2}{16\pi^2 L}\,, \tag{13}
$$

where $\zeta(3)$ is the Riemann-zeta function. So, the mean deviation from the classical propagation time due to the lightcone fluctuations is

$$\Delta t \approx \sqrt{\frac{\zeta(3)}{16\pi^2 L}} \sqrt{32\pi G_5}\, \rho \approx \left(\frac{r}{L}\right) t_{pl}\,. \tag{14}$$

Here we have used the fact that $G_5 = G_4\, L$, and $t_{pl} \approx 5.39 \times 10^{-44} s$ is the Planck time.

Here Δt increases linearly with the propagation distance, in contrast to the square root growth found in four dimensional compactified spacetime [4]. Equation (14) also reveals that the lightcone fluctuation effect is inversely related to L, the compactification length. This is due to the increased quantum fluctuations of fields confined in a smaller region. When r is of cosmological dimensions and L is sufficiently small, the effect is potentially observable.

Before we proceed further, it should be noted that we have set $\langle\sigma_1^2\rangle_R = 0$ when $L \to \infty$. This is the most natural choice of renormalization, corresponding to the effect of the graviton fluctuations vanishing in the limit of noncompactified spacetime. This is analogous to setting a Casimir energy density to zero in the limit of infinite plate separation. There is, however, another logical possibility. This is to set $\langle\sigma_1^2\rangle_R = 0$ at a finite value of L. If L is constant, then this procedure removes the lightcone fluctuations and sets $\Delta t = 0$. In our view, this is an unsatisfactory solution, as there seems to be nothing in the theory which picks out a particular finite value of L. In any case, changes in L would still be observable, as one could at most set $\langle\sigma_1^2\rangle_R = 0$ at one point along the path of a light ray. Let L_i be the compactification length at the time of emission, and $L_f = L_i(1+\delta)$, where $|\delta| \ll 1$, be that at the time of detection. Then the variation in flight times must satisfy [16]

$$\Delta t \gtrsim \sqrt{|\delta|} \left(\frac{r}{L}\right) t_{pl}\,. \tag{15}$$

The limits on the time variation of fundamental constants place various upper bounds [17,18] on $|\delta|$ in the range between 10^{-2} and 10^{-10}.

The variation in the flight time of pulses, Δt, can apply to the successive wave crests of a plane wave. This leads to a broadening of spectral lines from a distant source. Note, however, that Δt is the expected variation in the arrival times of two successive pulses only when they are uncorrelated. Following Ref. [12], we have examined [16] the correlation between two successive pulses separated in time by T, and found that if $r \gg T, L$, the two pulses are uncorrelated when $T \gg L^2/r$. Thus the pulses can be uncorrelated even when $T \ll L$. Now suppose that the experimental fractional resolution for a particular spectral line of period T is Γ. Then we must have $\Delta t/T \leq \Gamma$, which leads to a lower bound on L of

$$L \gtrsim \frac{r\, t_{pl}}{\Gamma T} = \frac{4 \times 10^4 cm}{\Gamma} \left(\frac{r}{1000 Mpc}\right) \left(\frac{E}{1 MeV}\right)\,, \tag{16}$$

where E is the energy of a photon with period T. This bound can be trusted only when the condition for uncorrelated pulses,

$$L \ll \sqrt{rT} = 6 \times 10^8 cm \left(\frac{r}{1000 Mpc}\right)^{\frac{1}{2}} \left(\frac{1 MeV}{E}\right)^{\frac{1}{2}}\,, \tag{17}$$

is satisfied.

The strongest lower bound would be deduced from the highest frequency spectral lines of the most distant sources, with the highest observed resolution. The best compromise

between these various requirements seems to be data from gamma ray burst sources, which involve cosmological distances and high frequencies, albeit low resolution. The use of gamma ray burst to constrain quantum gravity models was discussed by Amelino-Camelia, *et al* [19]. A typical burst is GRB990123, which involved [20] gamma rays at an energy of the order of $1\,MeV$ from a source with a redshift of at least $z = 1.6$. Assuming a Hubble constant of $H_0 = 65km/s/Mpc$ and a matter dominated universe, this corresponds to a distance of $r \gtrsim 2400\,Mpc$. If we take the resoluton Γ to be of order unity, this leads to a lower bound on L from Eq. (14) of $L \geq 10^5 cm$. This lower bound satisfies Eq. (17). This is a remarkably strong bound which would seem to rule out the five dimensional theory. Even if one adopts the approach of only using Eq. (15) to bound $|\delta|$, the result is $|\delta| \leq 10^{-10}(L/1\,cm)^2$, which for $L \ll 1cm$ is much stronger than the bounds cited above based upon time dependence of fundamental constants. It is of interest to note that reasonably strong bounds on L can be obtained from much lower frequency sources. Microwave lines from quasars [16], and the cosmic microwave background [21] both yield lower bounds on L of the order of a few tenths of a millimeter.

To conclude, we have demonstrated, in the case of one extra dimension, that the large quantum lightcone fluctuations due to the compactification of the extra dimension require the size of the extra dimension to be macroscopically large. This result seems to rule out the five dimensional Kaluza-Klein theory, or at the very least, place strong limits on the rate of change of the extra dimension. We must point out that the rate of growth of Δt with r depends crucially on the number and nature of the spatial dimensions. In four dimensions, $\Delta t \propto \sqrt{r/L}$, while in five dimensions $\Delta t \propto r/L$. We have also analyzed six and higher dimensional models in which the extra dimensions are flat. Here we find [16] that Δt grows only as a logarithmic function of r/L. In these models, data on spectral lines from distant sources yield no significant constraints on the compactification scale.

In principle, the effect discussed in this paper should be a generic phenomenon to be expected whenever there are small extra dimensions. This follows from the uncertainty principle argument given above, to the effect that confining the graviton modes in a small region of space should give rise to large fluctuations inversely related to the size of the region. However, there is a possibility of subtle cancellations making the net effect smaller than would naively be expected. This is what happens in the models with two or more flat extra dimensions. However, with two or more extra dimensions, it would also be possible to have the extra dimensions curved. The lightcone fluctuations in such models have not yet been examined. It is entirely possible that they might exhibit a rate of growth intermediate between linear and logarithmic functions. The phenomenon discussed in this paper can not only constrain models with extra dimensions, but could conceivably lead to positive confirmation of the existence of such dimensions.

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